**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

**Here *μ*** =**45 and *σ* = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour the work must be finished so we have X ≤ 50 . we have to find P(X > 50).**

**P(X > 50) = 1 - P(X ≤ 50).**

**Z = (X - *μ*)/ *σ*** **= (X - 45)/8**

**Using the Standard normal table**

**P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0) = P(Z ≤ 0.625)=73.4%**

**Probability that the service manager will not meet his demand will be = 1-0.734 = 0.26 or 26.76%**

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

**False.** If more employees are older than 44,then  *μ* will be around 44 and the given standard deviation is 6 ,considering this the mean cannot be more than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True.**

**Probabilty of employees less than age of 30 = P(X<30).**

**Z = (X -*μ* )/ *σ* = (30 - 38)/6**

**using the standard normal table**

**P(X ≤ 30) = P(Z ≤ (30 - 38)/6) = P(Z ≤ -1.333)=9.12%**

**So the number of employees with probability 0.912 of them being under age 30 is**

**P(X<30)=no:of favourable outcomes/total number of outcomes**

**0.0912\*400=36.4( or 36 employees).**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

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**As both are independent normal random variables, X1 + X2 is normal with N(µ1+µ2,** **σ12+** **σ22).**  **The variance will become 2 times more than the variance of X1 + X2.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**The Probability outside the a and b area is 0.01 (ie. 1-0.99)**

**Since we have the probabilities of a and b , we need to calculate X**

**The Probability towards left from a = -0.005 and right b = 0.005 (ie. 0.01/2)**

**For probability 0.005 the value of Z is -2.57**

**-(-2.57)\*20+100 = 151.4**

**(-2.57)\*20+100 = 48.6**

**Answer is option D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**norm.ppf (0.025,45\*5,3)**

**219.12**

**norm.ppf (0.975,45\*5,3)**

**230.87**

**norm.ppf (0.025,45\*7,3)**

**309.12**

**norm.ppf (0.975,45\*7,3)**

**320.87**

**So, the Rupee range with 95% probability for the annual profit of the company is given by,**

**=Profit1 + Profit2**

**= [219.12, 230.87] + [309.12, 320.87]**

**= [528.24, 551.74]**

**So Profit of the company in Rupees (in Million) is between range [528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**norm.ppf(0.05,45\*7,3)**

**310.065**

**norm.ppf(0.05,45\*5,3)**

**220.065**

**5th percentile of profit = 310.0654+ 220.0654 = 530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Division 2 with distribution N(7, 42)**